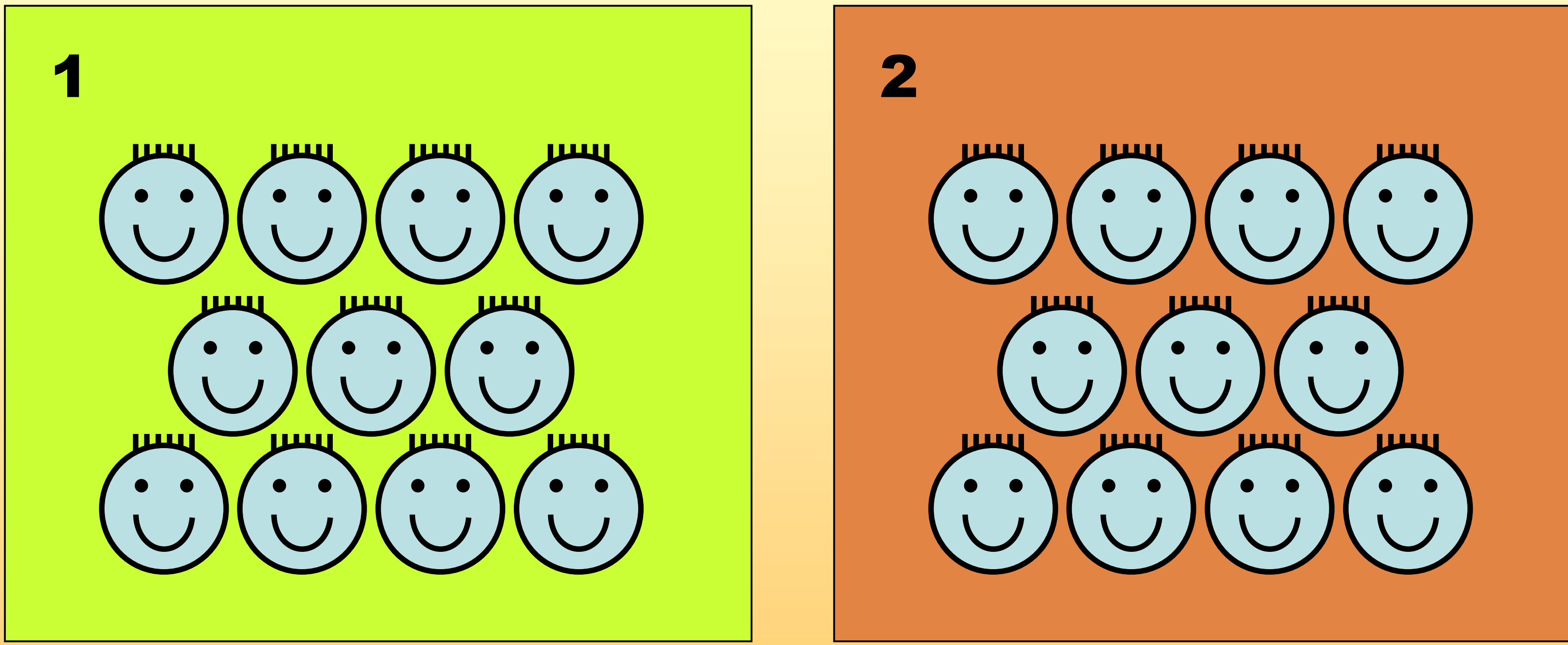
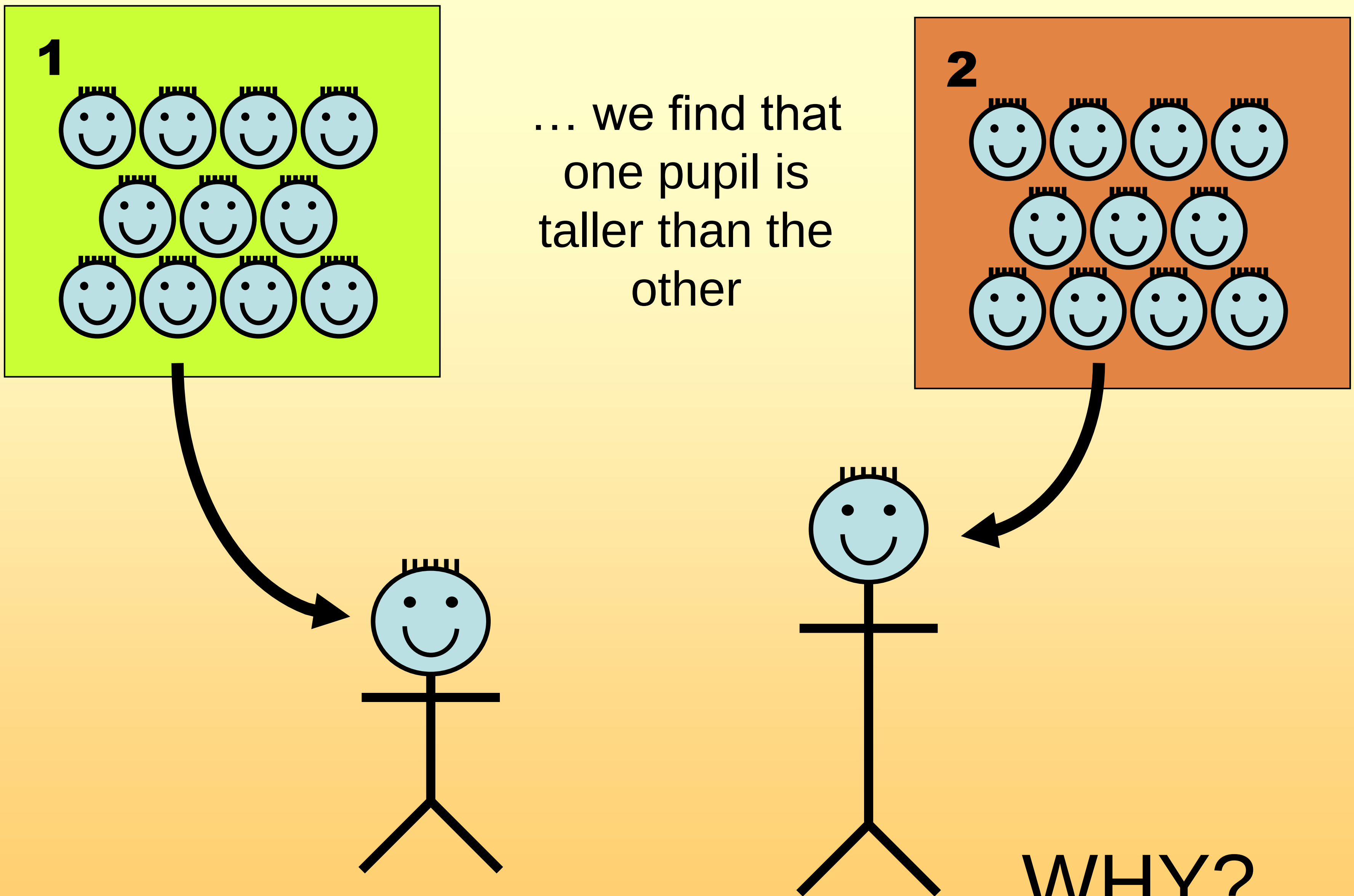
Are our results reliable enough to support a conclusion?

The T-Test

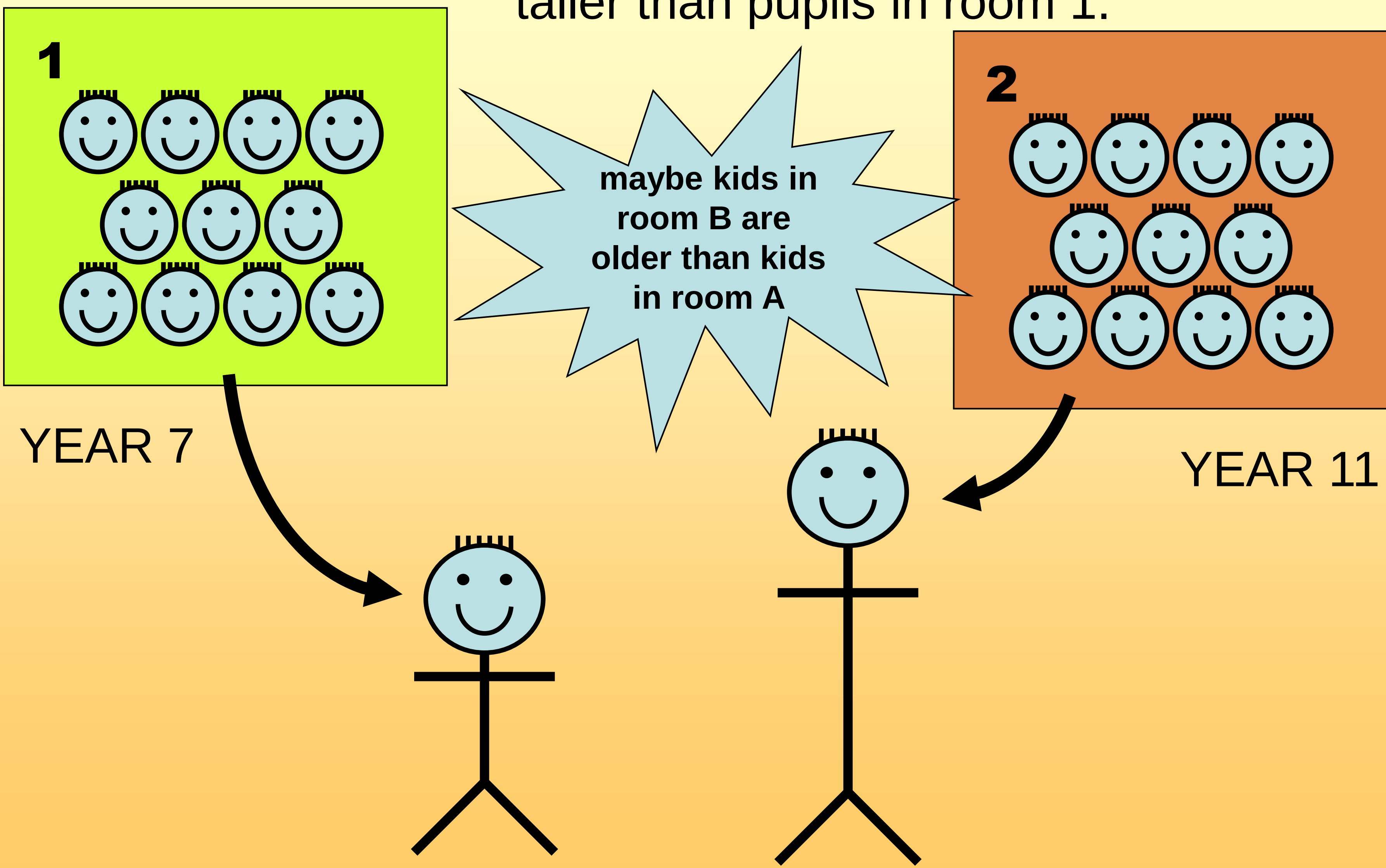
Imagine we chose two children at random from two class rooms…



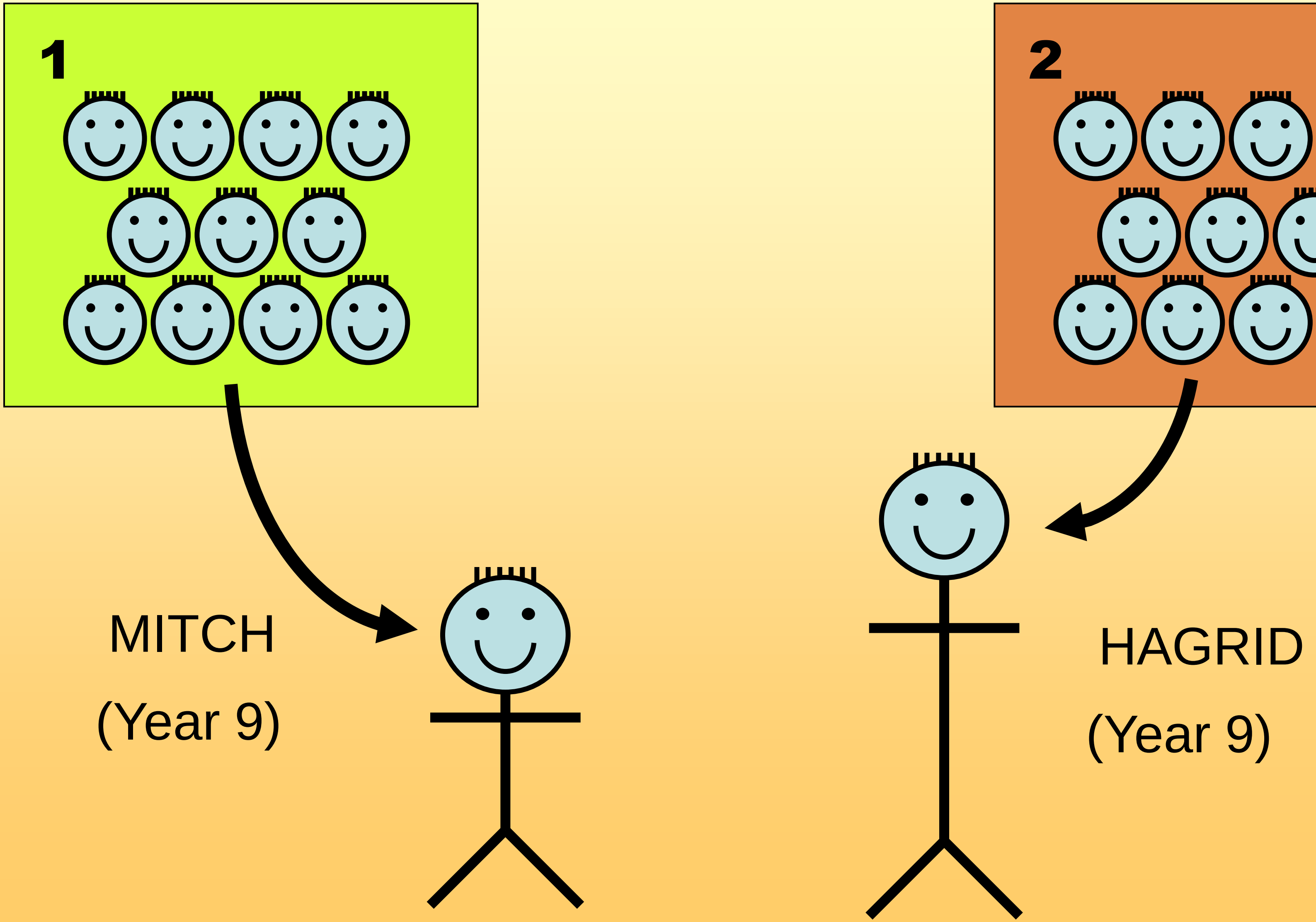
… and compare their height …


1
2
… we find that one pupil is taller than the other
WHY?

REASON 1: There is a significant difference between the two groups, and pupils in room 2 are actually taller than pupils in room 1.



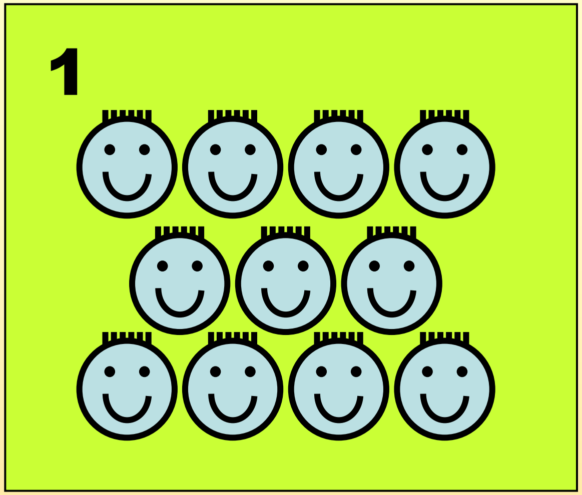
REASON 2: By chance, we picked a short pupil from room 1 and a tall one from room 2



How do we decide which reason is most likely?

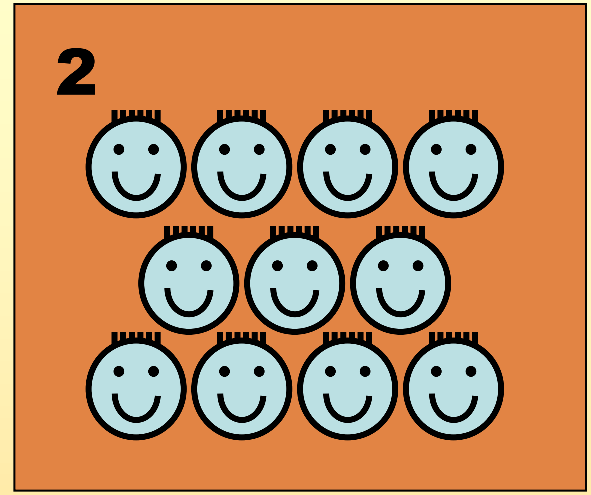
MEASURE MORE STUDENTS!!!

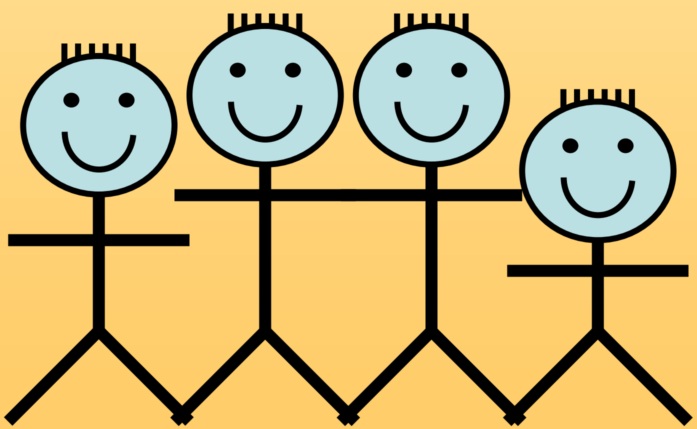
If there **is a significant difference** between the two groups…

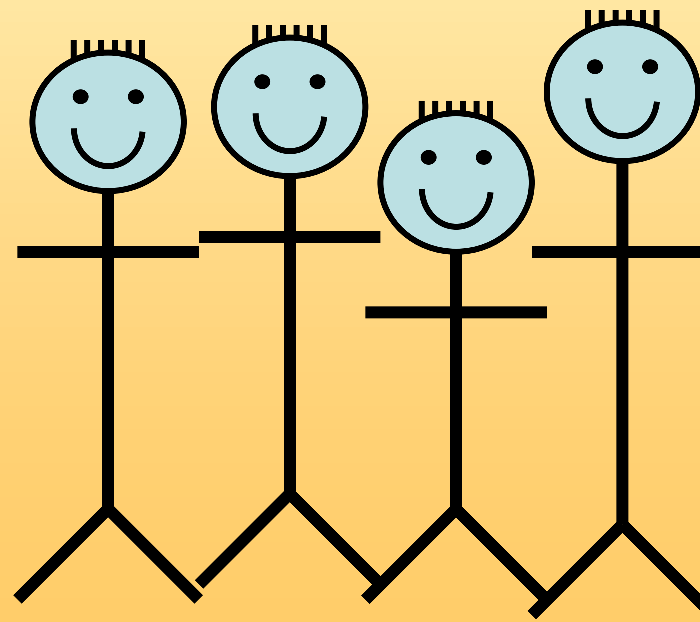


… the mean height of the two groups should be very…

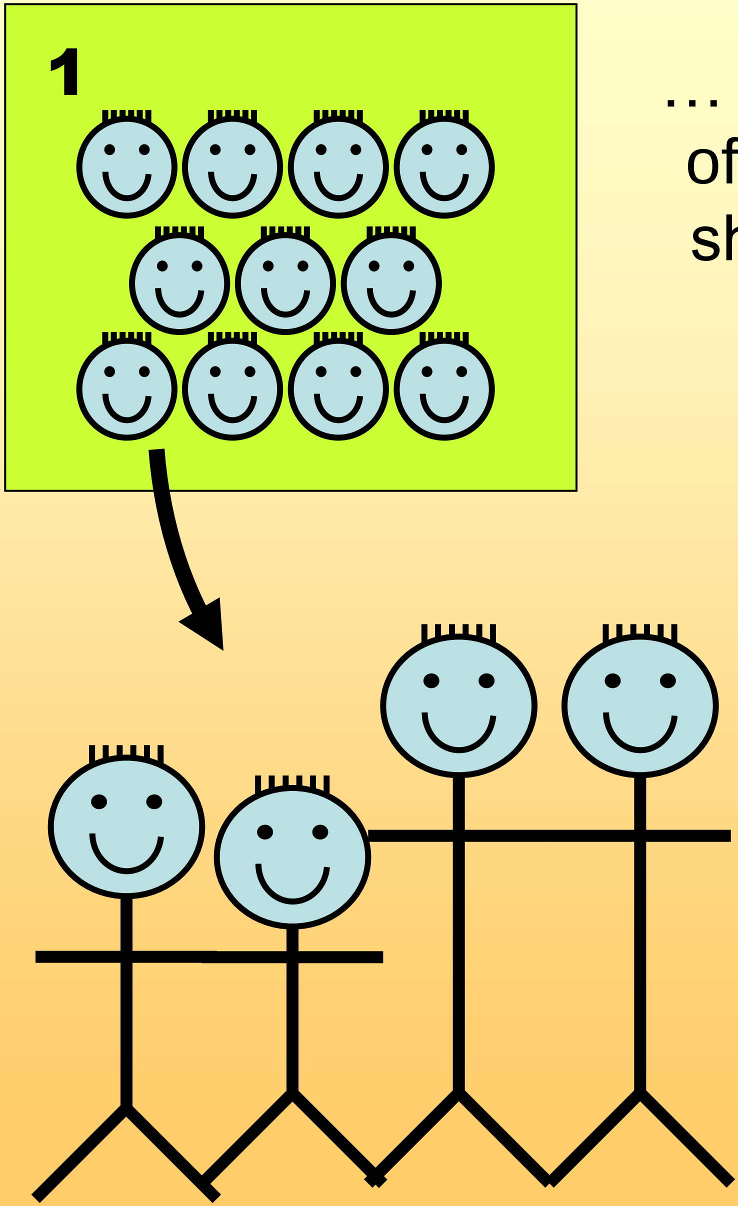
… DIFFERENT





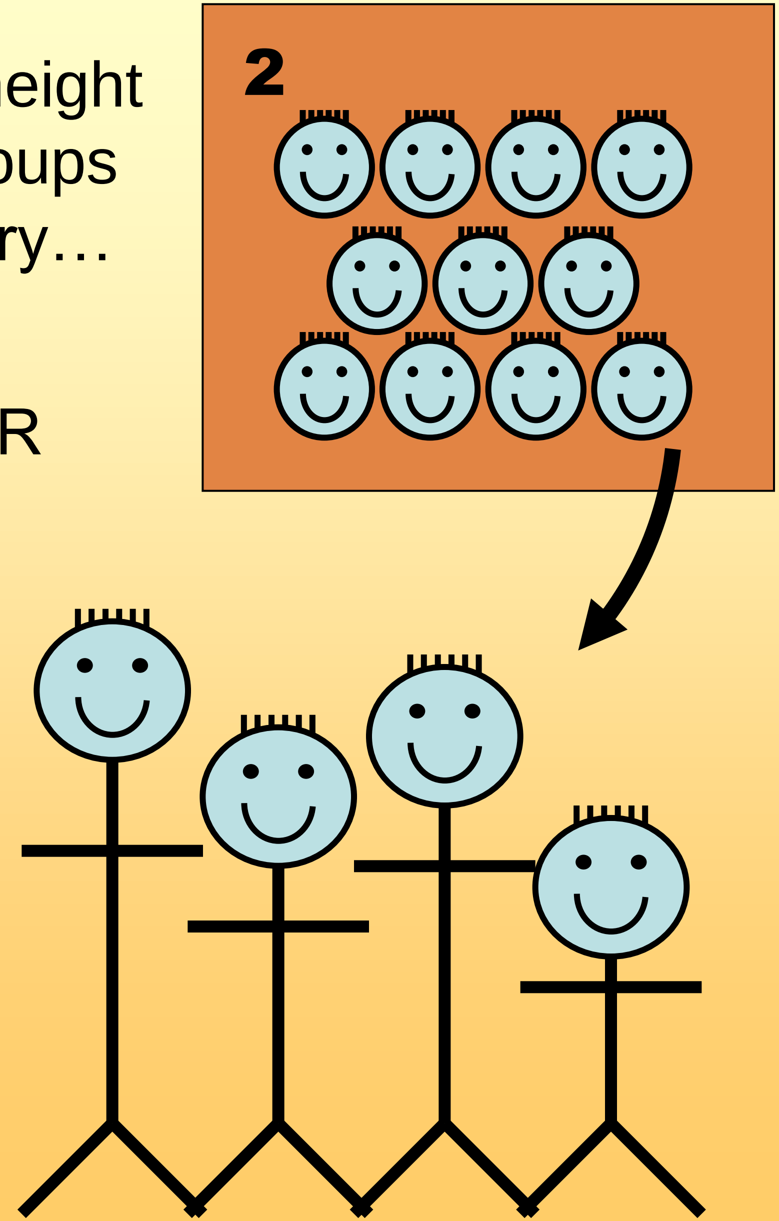


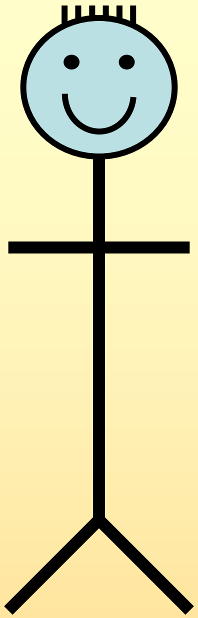
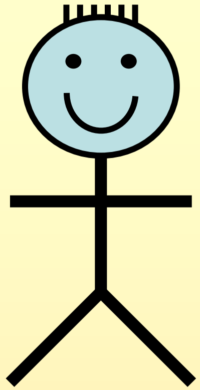
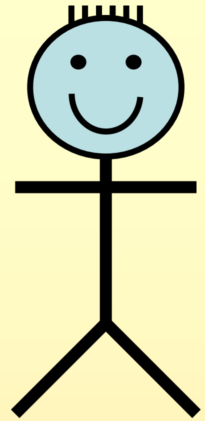
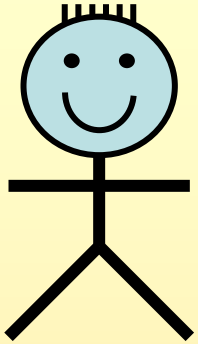
If there is **no significant difference** between the two groups…



… the <u>mean</u> height of the two groups should be very…

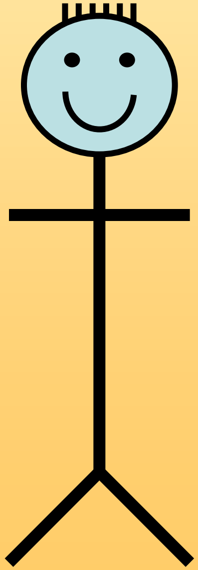
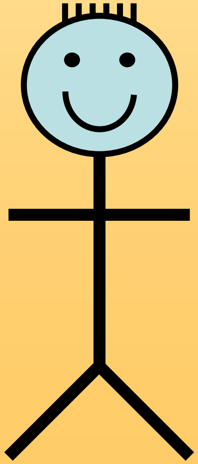
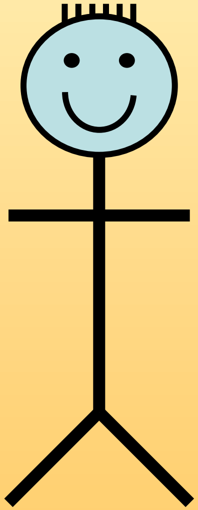
… SIMILAR



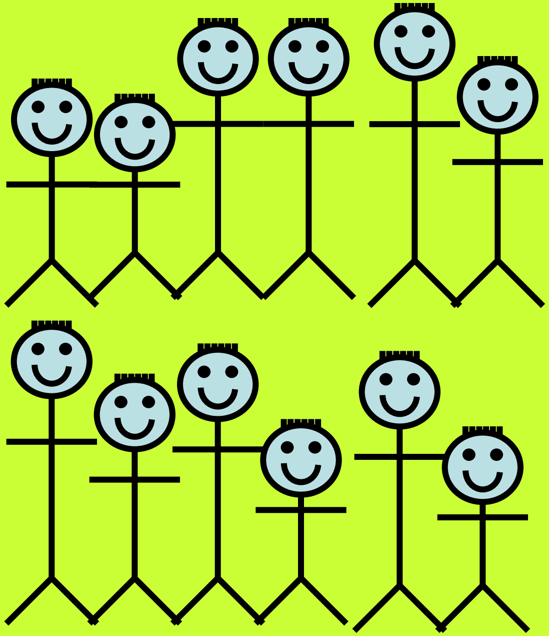
Remember:

Living things normally show a lot of variation, so…

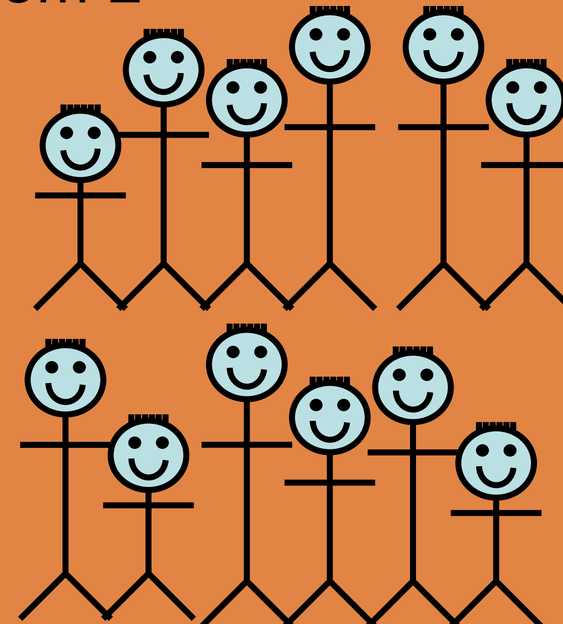
It is *VERY* unlikely that the mean height of our two samples will be exactly the same

Room 1



Average height = 162 cm

Room 2



Average height = 168 cm

Is the difference in average height of the samples large enough to be significant?

Student’s *t*-test

The Student’s *t*-test is a statistical test that compares the averages and standard deviations of two samples to see if there is a significant difference between them.

We start by calculating a number, *t*

t can be calculated using the equation:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

Where:

$\bar{x}_1$ is the mean of sample 1
s_1 is the standard deviation of sample 1
n_1 is the sample size of sample 1
$\bar{x}_2$ is the mean of sample 2
s_2 is the standard deviation of sample 2
n_2 is the sample size in sample 2

Worked Example: Random samples were taken of pupils in room 1 and room 2.

Their recorded heights are shown below…

	Students in Room 1					Students in Room 2				
Student Height (cm)	145	140	138	142	154	148	153	157	161	162
	154	158	160	166	166	162	163	167	172	172

	Students in Room 1					Students in Room 2				
Student Height (cm)	145	140	138	142	154	148	153	157	161	162
	154	158	160	166	166	162	163	167	172	172

Step 1: Calculate the mean height for each sample

$$t = \frac{\left|\bar{x}_1 - \bar{x}_2\right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Room 1: x_1 = **152.3 cm**

Room 2: x_2 = **161.7 cm**

Step 2: Find the absolute value of the difference between the means

$$t = \frac{\left| \bar{x}_1 - \bar{x}_2 \right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$x_2 - x_1 = 161.7 - 152.3 =$ **9.4**

Step 3: Work out the standard deviation for each sample

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Room 1: s_1 = **10.48** Room 2: s_2 = **7.66**

Step 4: Square the standard deviation for each group

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Room 1: s_1^2 = **109.79** Room 2: s_2^2 = **58.68**

Step 5:Divide each squared standard deviations by the sample size of that group.

$$t = \frac{\left| \bar{x}_1 - \bar{x}_2 \right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Room 1: 109.79 ÷ 10 = 10.98

Room 2: 58.68 ÷ 10 = 5.87

Step 6: Add these two values.

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

10.98 + 5.87= 16.85

Step 7: Take the square root of the number

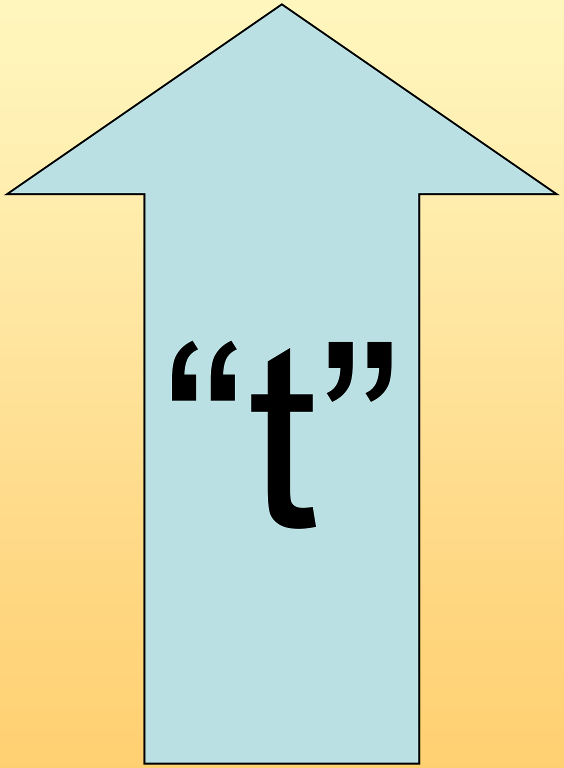
$$t = \frac{\left|\bar{x}_1 - \bar{x}_2\right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\sqrt{16.85} =$ 4.10

Step 8: divide the difference in the means (step 2) by the standard error of the difference (step 7)

$$t = \frac{\left| \bar{x}_1 - \bar{x}_2 \right|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

9.4 ÷ 4.10 = 2.28



Step 9: determine the degrees of freedom (df) for the test. In the t-test, the degrees of freedom is the sum of the sample sizes of both groups minus 2.

$(10 + 10) - 2 =$ **18**

"df"

Step 10: Given the df, look up the critical t-value in a standard table of significance

Use the 95% (p=0.05) confidence limit

df	.10	.05	.025	.01	.005	.000
1	3.078	[illegible]	12.706	31.821	63.657	636.619
2	1.886	[illegible]	4.303	6.965	9.925	31.598
[illegible]	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	**1.734**	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850

SO WHAT?

- **If your calculated t value is less than the number in the table, you conclude that the difference between the means for the two groups is NOT significantly different.**

- **If your calculated t value is greater than the number in the table, you conclude that the difference between the means for the two groups is significantly different.**

Calculated t-value = 2.28
Critical t- value = 1.734

Our calculated value of t *is above the critical value, therefore, there is a significant difference between the height of students in samples from room 1 and Room 2*

Do not worry if you do not understand *how* or *why* the test works

Follow the instructions CAREFULLY